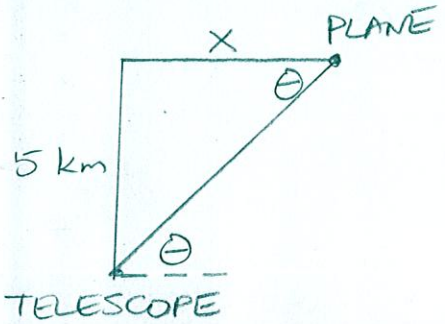
A plane flies horizontally at an altitude of 5 kilometers and passes directly over a tracking telescope on the ground. When the angle of elevation is 60°, this angle is decreasing at a rate of 30° per minute. How fast is the plane travelling at that time ?

SCORE: _____ / 14 PTS

You must state the units for the final answer. You do NOT need to show the units during the intermediate steps of your work.



$\frac{d\theta}{dt} = -\frac{\pi}{6}$ RADIANS / MINUTE (1)

IF YOU INDICATED θ WAS THE ANGLE IN THE DIAGRAM

WANT $\left.\frac{dx}{dt}\right|_{\theta = \frac{\pi}{3} \text{ RADIANS}}$

(1) IF YOU INDICATED X WAS THE DISTANCE IN THE DIAGRAM

$\cot\theta = \frac{x}{5\,km}$ (3) *

$-\csc^2\theta \frac{d\theta}{dt} = \frac{1}{5\,km}\frac{dx}{dt}$ (2)

(1) $\left(-\csc^2\frac{\pi}{3}\right)\left(-\frac{\pi}{6}\text{ RADIANS / MINUTE}\right) = \frac{1}{5\,km}\frac{dx}{dt}$ (2)

MUST BE IN RADIANS, NOT DEGREES

$(5\,km)\left(-\frac{4}{3}\right)\left(-\frac{\pi}{6}\text{ RADIANS / MINUTE}\right) = \frac{dx}{dt}$ (2)

$\frac{10\pi}{9}\frac{km}{min} = \frac{dx}{dt}$

(1) (1)

(*) OTHER SOLUTIONS POSSIBLE DEPENDING ON WHAT EQUATION YOU USED AT THIS STEP – TALK TO ME

Use an appropriate linear approximation to estimate $\sin^{-1} 0.48$.

SCORE: ____ / 6 PTS

$f(x) = \sin^{-1} x$ NEAR $x = \frac{1}{2}$

$f'(x) = \frac{1}{\sqrt{1-x^2}}$ (1)

$L(x) = f(\frac{1}{2}) + f'(\frac{1}{2})(x - \frac{1}{2})$

$= \frac{\pi}{6} + \frac{1}{\sqrt{1-\frac{1}{4}}}(x - \frac{1}{2})$

$= \frac{\pi}{6} + \frac{2}{\sqrt{3}}(x - \frac{1}{2})$ (1) (1) (1)

+(1) FOR USING $x = \frac{1}{2}$ AS POINT WHERE LINEAR APPROXIMATION IS BASED

$\sin^{-1} 0.48 \approx \frac{\pi}{6} + \frac{2}{\sqrt{3}}(0.48 - \frac{1}{2})$

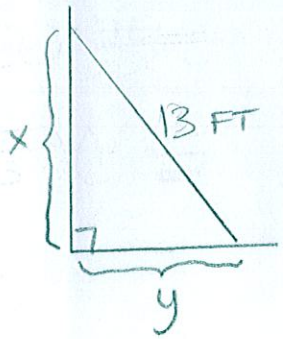
$= \frac{\pi}{6} + \frac{2}{\sqrt{3}}(-0.02)$

$= \frac{\pi}{6} - \frac{2}{\sqrt{3}}\frac{1}{50} = \frac{\pi}{6} - \frac{1}{25\sqrt{3}}$ (1)

A ladder 13 feet long rests against a vertical wall. If the bottom of the ladder is being pushed towards the wall at 2 feet per second, how fast is the top of the ladder moving against the wall when the bottom of the ladder is 5 feet from the wall ?

SCORE: ____ / 10 PTS

You must state the units for the final answer. You do NOT need to show the units during the intermediate steps of your work.



$\frac{dy}{dt} = -2\ \frac{ft}{sec}$ (1) IF YOU INDICATED y WAS THE DISTANCE IN THE DIAGRAM

WANT $\left.\frac{dx}{dt}\right|_{y=5\,ft}$ (1) IF YOU INDICATED x WAS THE DISTANCE IN THE DIAGRAM

$x^2 + y^2 = (13\,FT)^2 = 169\,FT^2$ (1)*

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ (2)

$2(12\,ft)\frac{dx}{dt} + 2(5\,ft)(-2\ \frac{ft}{sec}) = 0$ (1) (1) (1)

$\frac{dx}{dt} = \frac{10}{12}\ \frac{ft}{sec} = \frac{5}{6}\ \frac{ft}{sec}$ (1) (1)

(*) OTHER SOLUTIONS POSSIBLE DEPENDING ON WHAT EQUATION YOU USED AT THIS STEP - TALK TO ME